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Therefore—

$$a = 184^{\text{mm}}\cdot 3 = 6''\cdot 71.$$

Let us compare the geometrical elements of *Castor* thus obtained with those which are found by the authors of the "Handbook":—

Elements of the Handbook.	Glasenapp's Elements.
$a = 6''\cdot 67$	$a = 6''\cdot 71$
$e = 0\cdot 38$	$e = 0\cdot 38$
☊ $= 28^\circ\cdot 25$	☊ $= 28^\circ\cdot 18$
$i = 32\cdot 25$	$i = 32\cdot 30$
$\lambda = 305\cdot 17$	$\lambda = 305\cdot 35$

The elements T and n must be determined by one of the usual methods.

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On the Determination of Normal Places. By Lieut-General J. F. Tennant, R.E., F.R.S.

When it is proposed to correct an approximate orbit by comparing the places deduced from it with a large number of observations, and obtain corrections to the elements by the method of least squares, each observated element of the place would produce an equation of condition. It becomes then necessary to reduce the number of these equations of condition somehow, so as to limit the amount of computation in forming normal equations.

It seems to have been the practice to compute the equation of condition for each observation, and then to take the mean of the equations corresponding to a group of observations, and use it as one equation. Now the use of normal places formed by comparing the observations themselves with an ephemeris from approximate elements is practically universal. Of course the accuracy of these normals is vital. No care in computing can produce trustworthy results unless the normals are accurate, and to make them so a considerable number of observations have to be used, which probably extend over a considerable time. The determination of normal places has been treated of in the fourth volume of Oppolzer's *Lehrbuch*, but his remarks do not seem to me to quite meet the wants of those who have not all his experience and skill to guide their steps.

The difference between the series of places computed from the approximate elements and the true ones may be expressed by a series of the form

$$a + bt^2 + ct^2 + \&c.$$

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for a moderate interval of time. If t be the interval between the time for which a normal is desired and that of an observation, a will be the error of the ephemeris at the date of the normal; and the problem is to determine this with the greatest accuracy from a series of fallible places.

The date of the normal should be very near the mean of the dates of the observations employed in determining it. I shall assume that all the observations are of equal value, in which case we shall have the mean of observed errors of ephemeris

$$= a + \frac{\Sigma(t)}{n} b + \frac{\Sigma(t^2)}{n} c + \&c.$$

where n is the number of observations.* When the mean of the dates of observation is taken as that of the normal $\Sigma t = 0$ and $\Sigma(t^p)$ will be small when p is odd; also in all cases the values of b, c, &c. will rapidly decrease, so that we need never take into consideration more than three terms of the series; but it is evident that the value of c as deduced from the observations will largely depend on their errors. If the true value be insensible, then the value of a will be injuriously effected by determining c from the observations and by an amount which may be serious.

Oppolzer's suggestions practically come to these, that when the series of observations cannot be represented sufficiently by two terms of the series we should compare them with more approximate elements; or that, failing such, we should adopt a graphical construction; plot the errors as ordinates, with the time intervals as abscissæ; and, drawing an even curve through the points so as to represent their general run, use the ordinate corresponding to the date of observation as the error of ephemeris. I do not think this procedure will ever satisfy computers; if for no other reason, because the drawing of such an even curve satisfactorily requires an amount of technical skill which can only be attained by long practice; but otherwise the procedure is good, because it forces on one's mind the uncertainty of the result, which one is apt to forget when dealing with the arithmetical procedure, even if one does calculate its probable error.

I would suggest the following considerations as guides as to the use of the term involving t^2 in calculating normal places.

1st. Unless the rate of motion in one direction be rapidly changing, the use of the term involving t^2 will be objectionable.

2nd. If the course of the object in the neighbourhood of the normal place be nearly an arc of a great circle, it is highly probable that the use of the term in t^2 will be injurious.

3rd. If the course of the object be curved, then it may be dedesirable to determine c; but the first consideration will show for which element of observation it is necessary.

If the greatest accuracy be desirable, then, I think, the best plan would be to get in one of the above ways approximate errors

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* Computers will readily make the necessary changes if the observations are of varying weights.

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of ephemeris and get first corrections to the elements of the orbit. From these compute not only the places corresponding to the normal date, which are necessary for a further correction, but a place on each side of this, and from the differences between these places and those given by the original ephemeris deduce a value of c. If now we use this in the expression

$$a = \text{mean of errors of ephemeris} - \frac{\Sigma(t)}{n}b - \frac{\Sigma(t^2)}{n}c$$

instead of the value of c from the observations themselves, we shall get a good normal without the labour of computing an entire ephemeris and comparing it with observation.

Thus suppose we compute the place of the object for p days before and after the normal date as well as for that date, and the difference between the ephemeris and these revised places be on the three dates e_1, e_0, e_2, we may put

$$e_1 = a_1 - b_1 p + cp^2$$
$$e_0 = a_1$$
$$e_2 = a_1 + b_1 p + cp^2;$$

whence

$$\tfrac{1}{2}(e_1 + e_2) = a_1 + cp^2, \text{ and}$$
$$c = \frac{1}{p^2}\{\tfrac{1}{2}(e_1 + e_2) - e_0\}.$$

It is needless to compute b_1 and a_1 because an error in b will have little effect; and, moreover, owing to the change in the elements, b and b_1 would not probably be alike; but this value of c will be better than any value deduced directly from the observations.

Of course the corrected normal places would be used for the second correction of elements, and probably it would always suffice to ignore the value of e in getting places for the first investigation.

One advantage of this procedure would be that it would generally suffice to use three places of decimals in the equations of condition, and one less than Oppolzer uses all through the rest of the work, which, indeed, could now be carried out with the slide rule with much saving of labour.

On the Orbit of Comet I. (Sawerthal) of 1888. By Lieut-General J. F. Tennant, R.E., F.R.S.

This comet was discovered at the Cape of Good Hope on the morning of February 19, 1888, by Mr. Sawerthal of the Observatory. Observations were of course very frequent in the earlier part of its course, but fell off greatly as other new objects called for attention, though it continued visible as a faint object till the middle of August of that year.